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Comparison of Methods for Calculating Turbine Work in the Air Turborocket

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The air turborocket (ATR) is an airbreathing propulsion system that utilizes a turbine-drive gas source, which also provides fuel for the main combustor. By making some simplifying assumptions, ATR specific impulse becomes largely a function of turbine specific work and main-combustor gas total temperature. Turbine specific work is the major driver of ATR specific impulse because it also determines the main-combustor fuel-to-air ratio. Turbine specific work can be calculated assuming the turbine-drive gas is an equilibrium gas mixture, which expands without reaction, a nonreacting gas mixture, or an equilibrium gas mixture, which expands with reaction. Turbine work, main-combustor gas total temperature, and specific impulse based on these assumptions are compared. Significant errors in specific impulse result if an equilibrium, nonreacting gas mixture is assumed. This assumption can also lead to the mistaken conclusion that there are two maximum specific-impulse values at significantly different gas-generator oxidizer-to-fuel ratios. By assuming a reacting gas mixture, it is shown that the maximum specific-impulse of an O_2/H_2 driven ATR occurs at a gas-generator oxidizer-to-fuel ratio of about 4. There are two possible maximum specific-impulse values for the O_2/propane-driven ATR at gas-generator oxidizer-to-fuel ratios of about 1 and 2.

Nomenclature

Cp	=	specific heat at constant pressure, kJ/(kg-K)
Cpcomb	=	*Cp* of combustor gas, kJ/kg-K
CpTcomb	=	product of *Cp* and temperature of combustor gas, kJ/kg
CpTturb	=	product of *Cp* and temperature of turbine-drive gas, kJ/kg
Cpturb	=	*Cp* of turbine-drive gas, kJ/kg-K
DELTA *h*	=	change in enthalpy, kJ/kg
DELTA *h* turb	=	change of enthalpy of turbine-drive gas, kJ/kg
DELTA *h* turb-EQUILIBRIUM	=	change of enthalpy of turbine-drive gas assuming equilibrium composition, kJ/kg
DELTA *h* turb-FROZEN	=	change of enthalpy of turbine-drive gas assuming frozen (fixed) composition, kJ/kg
DELTA *h* turb-IDEAL	=	change of enthalpy of turbine-drive gas assuming constant *Cp* during expansion, kJ/kg
f	=	main-combustor fuel/air ratio
h	=	static enthalpy, kJ/kg
Pcomb	=	total combustor gas pressure, kPa
PR	=	pressure ratio
T	=	static temperature, K
Tgg	=	total gas generator gas temperature, K
w	=	specific work, kJ/kg
γ	=	ratio of specific heats
Δx	=	change in parameter x
η	=	efficiency

Subscripts

air	=	air
amb	=	ambient
comb	=	combustor
comp	=	compressor
exit	=	nozzle exit plane
turb	=	turbine
2	=	compressor inlet

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Introduction

THE air turborocket (ATR) is an airbreathing propulsion system that utilizes a turbocompressor similar to a turbojet but with a turbine-drive gas source, which does not use the airflow through the compressor. As shown in Fig. 1, the turbine-drive gas is supplied from a bipropellant gas generator, which operates independently of the airflow through the engine. Therefore, the ATR gas generator provides the turbine-drive gas for operation of the turbomachinery as well as providing fuel for combustion in the main combustor. The ATR can generate net positive thrust from sea-level static conditions to supersonic flight conditions without a booster propulsion system (see Refs. 1–10). In the ATR the total turbine inlet temperature is essentially unaffected by flight speed and can be kept at a relatively low value regardless of the air temperature at the compressor discharge because the air does not pass through the turbine. The compressor total discharge temperature then, rather than the total turbine inlet temperature, determines the maximum speed of the ATR. In the turbojet the total turbine inlet gas temperature is higher than the total compressor exit temperature because the air-fuel combustion process occurs upstream of the turbine, and thus the turbojet experiences a higher operating temperature (at the turbine inlet) than the ATR (at the compressor discharge) for the same flight condition. This higher operating total temperature enables the inherently higher maximum speed of the ATR. The ATR compressor provides a higher combustor pressure than that available from ram pressure alone at any given flight condition because of the compressor. Because the minimum combustor pressure is a major influence on the maximum altitude at which stable combustion can be maintained, the maximum operating altitude of the ATR is also inherently higher than that of the ramjet. The ATR main combustor pressure is also higher than the combustor pressure in a turbojet having the same compressor pressure ratio because the ATR compressor airflow goes directly to the combustor. Thus the ATR also has a higher maximum operating altitude than the turbojet. Although Fig. 1 shows a liquid-bipropellant gas-generator driven ATR, both solid-propellant and monopropellant gas generators can also be used. This research considers ATR designs using O_2/H_2 and O_2/propane gas generators.

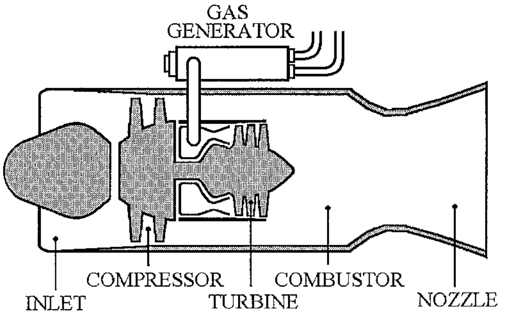

Fig. 1 Liquid bipropellant gas-generator ATR.

Calculation of ATR Specific Impulse

The delivered specific impulse for the ATR can be predicted using Eq. (1) (see Ref. 11 for detailed derivation), which includes the effects of ram drag and any pressure difference at the nozzle exit plane. Any thrust or specific-impulse degradation caused by external drag and inlet (additive) drag are not included.

$$I_{sp} = \left(1+\frac{1}{f}\right)\sqrt{2\eta_{comb}CpTcomb\left[1-\left(\frac{P_{exit}}{Pcomb}\right)^{(\gamma_{comb}-1)/\gamma_{comb}}\right]} + \frac{P_{exit}-P_{amb}}{\dot{m}_{turb}}A_{exit} - \frac{V_{air}}{f} \quad (1)$$

Equation (1) is valid at both design and off-design conditions and shows how ATR specific impulse varies with f (combustor fuel/air ratio) and other component parameters (combustor gas total temperature, nozzle exit pressure, etc.). It assumes that the main combustor gas behaves as an ideal gas with a constant *Cp*. This is a generally valid assumption because, if the combustor operates very fuel lean, the combustor gas will be largely air. At the anticipated temperatures and pressures in the main combustor, air behaves as an ideal gas. The first term in Eq. (1) represents the specific impulse that would result if the combustor gas is expanded isentropically and optimally from the combustor plenum (at a total pressure of *Pcomb*) to the nozzle static exit pressure (P_{exit}). The equilibrium (maximum) combustor gas temperature (*Tcomb*) is multiplied by the combustor efficiency (η_{comb}) to predict the actual combustor gas total temperature. The $[1+(1/f)]$ term is actually the ratio of total combustor flow rate to fuel flow rate. It appears in the equation because the airflow rate is not counted as part of the propellant flow rate in the determination of an airbreathing engine specific impulse. This is fundamentally different from rocket engines in which both the fuel and oxidizer are part of the propellant flow rate that is used to calculate specific-impulse. The $[1+(1/f)]$ term varies nonlinearly and inversely with f. Thus as f decreases, the ratio of total combustor flow rate to fuel flow rate increases, which increases specific impulse. Similarly, as f increases, the ratio of total combustor flow rate to fuel flow rate decreases, which reduces specific impulse. The second term represents the increase (or decrease) in specific impulse as a result of the ambient-to-exit pressure difference, which acts over the nozzle exit area (A_{exit}). This term will be nonzero for any nonoptimum gas expansion in the nozzle. This second term usually contributes less to the specific impulse than the first term. The last term represents the loss in specific impulse as a result of ram drag. It increases as vehicle velocity (or V_{air}, inlet air velocity) increases but decreases as f increases.

Before simplifying the preceding expression, it is assumed that the inlet is sized so that additive drag at the engine design point is zero. The pressure thrust acting over the nozzle exit area is assumed to be significantly less than the thrust caused by the velocity of the combustor gas expanded in the main nozzle. Finally, ram drag is neglected mainly because gross thrust is of more concern for the purposes of this research. At the same time it is recognized that ram drag becomes significant as f is reduced and/or flight velocity increases. With these simplifying assumptions the predicted specific impulse for gross thrust becomes

$$Isp \approx (1+1/f)\sqrt{2\eta_{comb}CpTcomb\left[1-(P_{exit}/Pcomb)^{(\gamma_{comb}-1)/\gamma_{comb}}\right]} \quad (2)$$

The main combustor gas properties (*Cpcomb* and *Tcomb*) are very strong functions of f. Equation (2) indicates that specific impulse will increase as f is reduced or *CpTcomb* is increased. However, f and *CpTcomb* are related. Specifically, in the fuel-rich regime *CpTcomb* increases as f is reduced, thus increasing specific impulse. The assumption is made here that *CpTcomb* varies closely with *Tcomb* because *Cpcomb* does not vary significantly with f. In the fuel-lean regime f and *CpTcomb* vary together. Thus reducing f in this case should improve specific impulse, whereas the resulting reduction in *CpTcomb* tends to reduce specific impulse. An earlier study[11] determined analytically that the net effect of reducing f in the fuel-lean regime of ATR operation is improved specific impulse.

For a fixed flight condition all of the combustor gas properties (ratio of specific heats, temperature, and molecular weight) appearing in Eq. (2) are weak functions of f. That is, if the combustor operates relatively fuel lean, the combustor gas can be modeled as hot air, which behaves ideally in the temperature and pressure regime of the ATR combustor. Specifically, the enthalpy change of the gas as it expands out of the nozzle can be equated to *Cp* of the air in the combustor plenum multiplied by the isentropic temperature decrease of the air corresponding to the expansion process from combustor plenum pressure to nozzle exit pressure. For the purposes of this research, the ratio of specific heats (γ_{comb}) of the combustor gas is assumed constant. If a compressor pressure ratio and compressor exit pressure (and hence *Pcomb*) are assumed, then the specific impulse can be approximated as

$$Isp = (1+1/f)C_1\sqrt{CpTcomb} \quad (3)$$

where C_1 is defined from

$$C_1 = \sqrt{2\eta_{comb}\left[1-(P_{exit}/Pcomb)^{(\gamma_{comb}-1)/\gamma_{comb}}\right]} \quad (4)$$

This expression shows that C_1 and specific impulse are reduced as the ratio of total combustor to nozzle exit pressure (nozzle pressure ratio) drops. The nozzle exit pressure will be substantially less than the total pressure because of the high velocity at the nozzle exit.

The nozzle pressure ratio has a significant influence on specific impulse especially as it approaches the critical value (about 1.9) below which supersonic flow from the main combustor nozzle is not possible. The selected nozzle-area ratios provide underexpanded flow in the nozzle. The corresponding nozzle-pressure ratios are less than the combustor-to-ambient-pressure ratio. For the Mach 2.0, 6000-m-altitude design the nozzle-pressure ratio is 9 to 1, and the nozzle-area ratio is 2 to 1. For the Mach 3.0, 9000-m-altitude design the nozzle-pressure ratio is 32 to 1, and the nozzle-area ratio is 3.9 to 1. The compressor ratios for the two designs are 3 and 2, respectively, and the resulting combustor pressures for the two designs are 1020 and 1906 kPa, respectively.

The last parameter required to calculate ATR specific impulse is f. This parameter is determined from a power balance between the compressor and turbine. Required compressor specific work can be developed by application of the first law of thermodynamics. By assuming that air is calorically perfect, the enthalpy difference the air experiences as it flows through the compressor is equal to *Cp* multiplied by the temperature change. Because of irreversibilities in the compression process, the actual specific work required to operate the compressor is always more than the minimum (isentropic) specific work. This additional work can be accounted for by the use of a compressor efficiency. The resulting expression is

$$w_{comp} = \eta_{comp}T_2Cp_{air}\left(PR_{comp}^{(\gamma_{air}-1)/\gamma_{air}} - 1\right) \quad (5)$$

Calculation of turbine specific work also comes from application of the first law of thermodynamics. By making similar assumptions, except for that of an ideal gas, the maximum turbine specific work is

$$w_{\text{turb MAXIMUM}} = \Delta h_{\text{turb}} \tag{6}$$

where Δh is the change in enthalpy of the gas, or gas mixture, as it flows through the turbine. The maximum enthalpy change is only possible if the gas expands isentropically from the actual turbine inlet pressure to the actual turbine exit static pressure. Turbine efficiency (η_{turbine}) is used to account for this less-than-maximum turbine work:

$$w_{\text{turb}} = \eta_{\text{turb}} \Delta h_{\text{turb}} \tag{7}$$

If the turbine-drive gas velocity change is significant through the turbine, then the total enthalpy change must be used instead of the static enthalpy change. By equating the turbine and compressor specific work, the following expression for f results:

$$f = \frac{\dot{m}_{\text{turb}}}{\dot{m}_{\text{air}}} = \frac{Cp_{\text{air}} T_2 \left(PR_{\text{comp}}^{(\gamma_{\text{air}}-1)/\gamma_{\text{air}}} - 1\right)}{\eta_{\text{comp}} \eta_{\text{turb}} \Delta h_{\text{turb}}} \tag{8}$$

The following sections discuss three ways of determining the Δh_{turb}.

Turbine Specific Work Assuming an Equilibrium, Nonreacting Drive Gas

For an ideal gas the relationship between temperature and pressure for isentropic flow is

$$T_1/T_2 = (P_1/P_2)^{(\gamma-1)/\gamma} \tag{9}$$

For application to turbines, the subscript 1 refers to the turbine inlet condition and 2 refers to the turbine exit condition. Thus P_1/P_2 is the actual turbine-pressure ratio, whereas T_1/T_2 is the (maximum) total temperature ratio that would result if the expansion process in the turbine was isentropic. The preceding expression can be used to calculate this maximum total temperature drop from

$$\Delta T = T_1 - T_2 = T_1(1 - T_2/T_1) = T_1\left[1 - (P_2/P_1)^{(\gamma-1)/\gamma}\right] \tag{10}$$

The maximum enthalpy change corresponds to this maximum decrease in temperature because the enthalpy of an ideal gas is a function of temperature only. Again, this assumes a negligible change in flow velocity through the turbine, which would require the use of total temperature and pressure. The actual temperature decrease in a turbine will be less than the predicted maximum because of irreversibilities unless there is significant heat gain from the surroundings. Therefore, the actual enthalpy decrease is always less than the maximum enthalpy decrease. Consequently, the turbine never produces the maximum possible (isentropic) work. Equations (7) and (10) can be combined to define Eq. (11), which predicts the maximum specific turbine work assuming an ideal (constant specific heat ratio), nonreacting turbine-drive gas and a known turbine-pressure ratio:

$$w_{\text{turb}} = \eta_{\text{turb}} Cpturb \Delta T_{\text{MAXIMUM}}$$

$$= \eta_{\text{turb}} Cpturb\, Tgg\left[1 - (1/PR_{\text{turb}})^{(\gamma_{\text{turb}}-1)/\gamma_{\text{turb}}}\right] \tag{11}$$

PR_{turb} is the actual pressure ratio (total-to-static) of the gas as it flows through the turbine, Tgg is the total turbine inlet temperature, nominally equal to the gas-generator exhaust gas total temperature, γ_{turb} is the ratio of specific heats of the ideal turbine-drive gas, and $Cpturb$ is the specific heat of the equilibrium gas mixture in the gas-generator plenum. The enthalpy decrease for the turbine-drive gas can then be defined as

$$\Delta h_{\text{turb-IDEAL}} = \eta_{\text{turb}} Cpturb\, Tgg\left[1 - (1/PR_{\text{turb}})^{(\gamma_{\text{turb}}-1)/\gamma_{\text{turb}}}\right] \tag{12}$$

The NASA John H. Glenn Research Center at Lewis Field equilibrium code, called CEA for chemical equilibrium and applications, was used to generate the gas properties including temperature, ratio of specific heat Cp, and enthalpy as function of expansion pressure ratio. The CEA code uses the minimization-of-free-enrgy method to determine chemical equilibrium. This method is described in detail in Chapter 2 of Ref. 12.

If the design point compressor and turbine characteristics (PR_{comp}, PR_{turb}, η_{comp}, and η_{turb}) and design point flight condition are known, then f can be defined from

$$f = C_2/\Delta h_{\text{turb-IDEAL}} \tag{13}$$

where

$$C_2 = \frac{Cp_{\text{air}} T_2 \left(PR_{\text{comp}}^{(\gamma_{\text{air}}-1)/\gamma_{\text{air}}} - 1\right)}{\eta_{\text{comp}}} \tag{14}$$

ATR specific impulse is then calculated from Eq. (3), which becomes

$$Isp = C_1(1 + \Delta h_{\text{turb-IDEAL}}/C_2)\sqrt{CpTcomb} \tag{15}$$

This expression shows that ATR specific impulse will increase as the enthalpy drop through the turbine increases, unless the combustor gas total temperature drops significantly.

All of the foregoing calculations, using data generated by the CEA code, were used to generate predicted ATR specific impulse values as a function of gas-generator O/F for O_2/H_2 and O_2/propane-powered ATRs.

Figure 2 is a plot of $\Delta h_{\text{turb-IDEAL}}$ (labeled DELTA h turb-IDEAL in figure) and $CpTcomb$ as a function of gas-generator oxidizer-to-fuel ratio (O/F) for an O_2/H_2-powered ATR. Figure 2 shows that $\Delta h_{\text{turb-IDEAL}}$ maximizes at an O/F of about seven, whereas $CpTcomb$ maximizes at an O/F of 1 or 2 depending on the ATR design. Thus, $\Delta h_{\text{turb-IDEAL}}$ maximizes at a slightly fuel-rich condition in the gas generator. It is not apparent from these data alone which of the two parameters has the greater influence on ATR specific impulse; however, they do suggest the possibility of more than one relative maximum value of specific impulse as a function of gas-generator O/F.

A plot of the same data for an O_2/propane-powered ATR is shown in Fig. 3. In this case $\Delta h_{\text{turb-IDEAL}}$ maximizes at O/F values of about

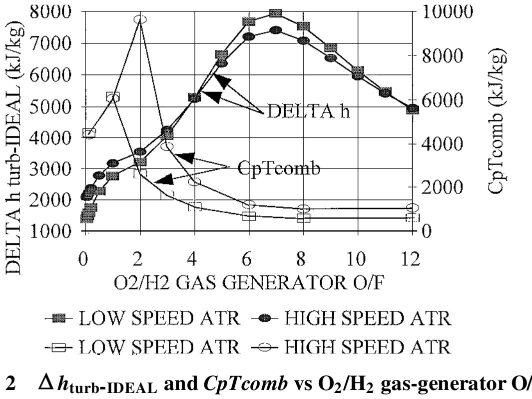

Fig. 2 $\Delta h_{\text{turb-IDEAL}}$ and $CpTcomb$ vs O_2/H_2 gas-generator O/F.

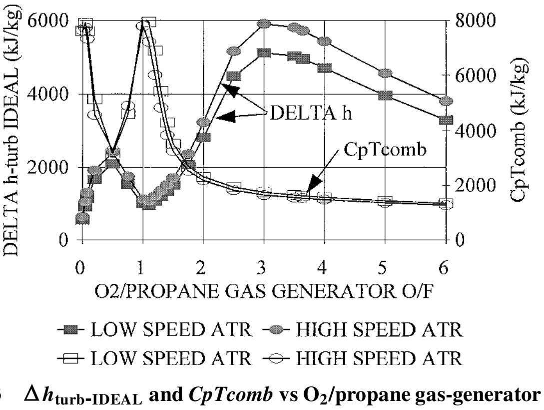

Fig. 3 $\Delta h_{\text{turb-IDEAL}}$ and $CpTcomb$ vs O_2/propane gas-generator O/F.

0.5 and 3, whereas *CpTcomb* maximizes at O/F values of nearly 0 and 1. The maximum *CpTcomb* value at the nearly zero gas generator O/F is not considered important in this study because an all-propane gas generator would not be a viable turbine-drive gas source for the ATR unless heated to a high temperature by an external source. The stoichiometric O/F of an O_2/propane gas generator is 3.64. Thus $\Delta h_{\text{turb-IDEAL}}$ maximizes at a slightly fuel-rich condition in the gas generator (O/F = 3) in addition to a very fuel-rich gas-generator condition (O/F = 0.5). Again, it is not possible, using this graph alone, to determine which parameter has the greatest effect on ATR specific impulse, but the data again suggest the possibility of two relative maximum ATR specific-impulse values.

The major conclusions of these data are the following: 1) $\Delta h_{\text{turb-IDEAL}}$ can be an important propellant-based parameter for evaluating ATR propellants because it directly affects both f and *CpTcomb*, both of which affect specific impulse, and 2) there appears to be the potential for two relative maximum specific-impulse values. In particular, the maximum $\Delta h_{\text{turb-IDEAL}}$ occurring at a O_2/propane gas-generator O/F of 0.5 is of considerable interest because it introduces the possibility of operating the turbine at relatively cool (fuel-rich) conditions greatly extending the life of the turbine without sacrificing specific impulse.

The variation of $\Delta h_{\text{turb-IDEAL}}$ with gas-generator O/F is determined largely by the variation of *Cpturb* and/or *Tgg* because the other factors that determine $\Delta h_{\text{turb-IDEAL}}$ are constant (turbine efficiency and pressure ratio) or nearly constant (γ_{turb}). *Cpturb* and *Tgg* are plotted separately as functions of gas-generator O/F in Figs. 4 and 5. These plots show, in the case of the O_2/propane gas generator, that the $\Delta h_{\text{turb-IDEAL}}$ in Figs. 2 and 3 is high at the fuel-rich condition because of a high *Cpturb* and not because of *Tgg*. That is, *Tgg* has a single maximum value at the stoichiometric condition and decreases from this value when the gas generator is either fuel rich or oxidizer rich. The *Cpturb* for the O_2/propane gas generator, on the other hand, has two distinct relative maximum values at O/F values of 0.5 and 3. *Cpturb* is for equilibrium gas conditions in the gas generator. Thus, the data in Figs. 2 and 3 indicating the possibility of a high specific impulse at fuel-rich gas-generator conditions are dependent on the value of *Cpturb*. Because of the potential importance of this second maximum specific-impulse value, *Cpturb* was examined more closely.

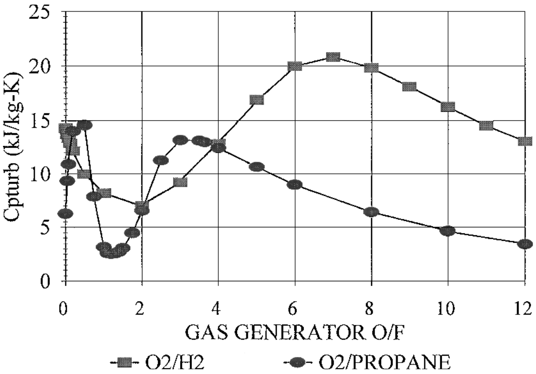

Fig. 4 Cp_{turb} vs gas-generator O/F.

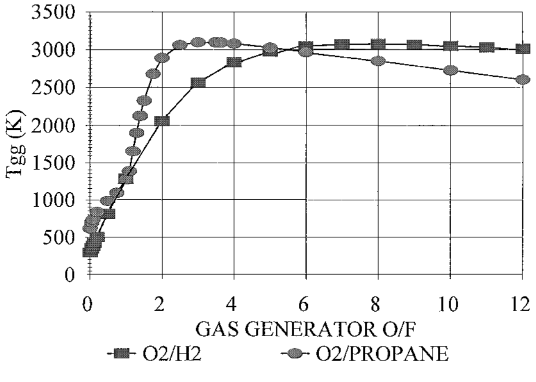

Fig. 5 *Tgg* vs gas-generator O/F.

Turbine-Drive Gas Ratio of Specific Heat Ratio at Constant Pressure (*Cp*)

Cp is defined for both ideal and real gases from

$$Cp = \left(\frac{\partial h}{\partial T}\right)_P \tag{16}$$

where subscript P specifies that *Cp* is the variation of enthalpy with respect to temperature for a constant pressure process. The enthalpy of a mixture of gases is defined as

$$h = \sum c_i h_i \tag{17}$$

where i is the number of constituents in the gas mixture, c_i is the mass fraction of the ith constituent, and h_i is the enthalpy of the ith constituent. Note these results taking the partial derivative of Eq. (16) using the expression from Eq. (17) as shown in Eq. (18):

$$Cp = \left(\frac{\partial h}{\partial T}\right)_P = \frac{\partial}{\partial T}\left(\sum c_i h_i\right)_P = \sum \frac{\partial}{\partial T}(c_i h_i)_P$$
$$= \sum\left(c_i\frac{\partial h_i}{\partial T} + h_i\frac{\partial c_i}{\partial T}\right)_P = \sum c_i Cp_i + \sum h_i\frac{\partial c_i}{\partial T} \tag{18}$$

This expression shows how the *Cp* of a gas mixture can change: 1) the *Cp* of one or more constituents can change as a result of temperature variations (first term) and/or 2) the fraction of one or more constituents can change as a result of temperature variations (second term). If the gas mixture is not reacting then, by definition, the constituent fractions are constant, and the second term disappears with the result that

$$Cp = \sum c_i Cp_i \tag{19}$$

In this case calculating *Cp* of a gas mixture requires adding, on a mole or mass fraction basis, the *Cp* of each constituent. The second term only becomes significant if the constituent fractions are sensitive to temperature variations. When the constituent mole or mass fractions are varying, this, by definition, is a reacting flow. The combustion products of O_2/H_2 and O_2/hydrocarbons, especially at relatively fuel-rich conditions, are reacting flows. This explains why the *Cp* of both the O_2/H_2 and O_2/propane combustion products experience a relative maximum value at very fuel-rich conditions. A further discussion of these two terms in Eq. (18) is found in Ref. 13 (pp. 530–533). As noted in Eq. (7), the specific work of a turbine is the decrease in enthalpy of the gas, ideal or otherwise, flowing through it. But $\Delta h_{\text{turb-IDEAL}}$ is not an accurate predictor of the actual enthalpy decrease of the turbine drive gas because *Cp* of a reacting gas varies significantly. Therefore, it cannot be used to predict turbine work or ATR specific impulse.

Turbine Specific Work Assuming a Reacting Equilibrium Gas

A second approach to predicting the turbine-drive gas enthalpy decrease is to assume that the gas mixture is at equilibrium throughout the expansion process (equilibrium flow). In this approach the enthalpy values generated by the CEA code are used directly to calculate the enthalpy decrease, which is then multiplied by the turbine efficiency. With this assumption the expression for ATR combustor fuel-to-air ratio becomes

$$f = \frac{Cp_{\text{air}}T_2\left(PR_{\text{comp}}^{(\gamma_{\text{air}}-1)/\gamma_{\text{air}}} - 1\right)}{\eta_{\text{comp}}\eta_{\text{turb}}\Delta h_{\text{turb}}} = \frac{C_2}{\eta_{\text{turb}}\Delta h_{\text{turb}}} \tag{20}$$

ATR specific impulse can now be calculated using Eq. (3), which becomes

$$Isp = C_1(1 + \Delta h_{\text{turb-EQUILIBRIUM}}/C_2)\sqrt{CpTcomb} \tag{21}$$

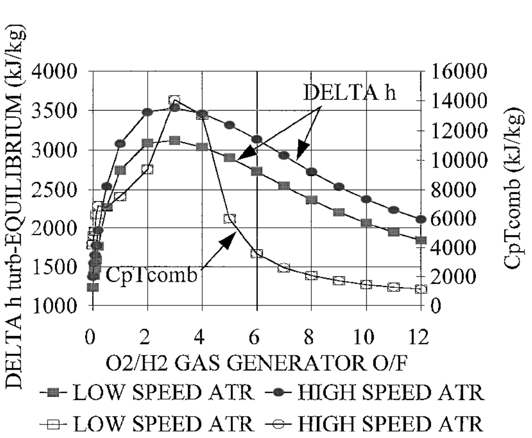

Fig. 6 $\Delta h_{turb\text{-}EQUILIBRIUM}$ and *CpTcomb* vs O_2/H_2 gas-generator O/F.

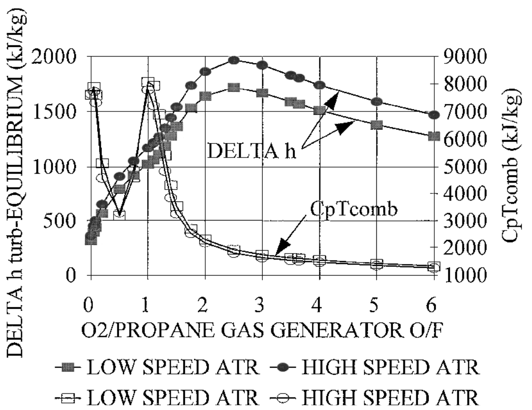

Fig. 7 $\Delta h_{turb\text{-}EQUILIBRIUM}$ and *CpTcomb* vs O_2/propane gas-generator O/F.

The resulting $\Delta h_{turb\text{-}EQUILIBRIUM}$ (labeled DELTA h turb-EQUILIBRIUM in figure) and corresponding *CpTcomb* are plotted as a function of gas-generator O/F in Fig. 6 for an O_2/H_2 gas generator. This figure shows that both parameters maximize at a gas-generator O/F of about 3. On the basis of this result, it is expected that the specific impulse will maximize at an O/F of about 3. Comparing $\Delta h_{turb\text{-}EQUILIBRIUM}$ to $\Delta h_{turb\text{-}IDEAL}$ plotted in Fig. 2 shows that at very low (less than about 1) and very high (above 40) O/F values, the two Δh_{turb} values are nearly equal. At either the very fuel-rich or very oxidizer-rich condition the turbine-drive gas is mostly hydrogen or oxygen and therefore behaves ideally. As soon as significant chemical reaction occurs, this assumption is no longer valid, and a significant difference between the two Δh_{turb} values results. Comparison of Figs. 2 and 6 indicates that the use of the ideal gas assumption with O/F values between 1 and 40 for O_2/H_2 can lead to major errors in predicting the maximum available turbine specific work.

A similar plot for the O_2/propane-powered ATR is shown in Fig. 7, where $\Delta h_{turb\text{-}EQUILIBRIUM}$ (labeled DELTA h turb-EQUILIBRIUM in figure) and *CpTcomb* are plotted as a function of gas-generator O/F. $\Delta h_{turb\text{-}EQUILIBRIUM}$ maximizes at a single gas-generator O/F of about 2.5. The maximum $\Delta h_{turb\text{-}IDEAL}$ values occur at O/F values of 0.5 and 3 in Fig. 3. The maximum values of *CpTcomb* occur at O/F values of nearly 0 and about 1. This difference in O/F values at which $\Delta h_{turb\text{-}EQUILIBRIUM}$ and *CpTcomb* maximize again indicates the possibility of two relative maximum specific-impulse values depending on the relative influence of these parameters on specific impulse. Comparing Figs. 7 and 3 also shows that $\Delta h_{turb\text{-}IDEAL}$ is significantly greater than $\Delta h_{turb\text{-}EQUILIBRIUM}$ for nearly all O/F values. Only at an O/F of about 1 will both assumptions yield similar values. This comparison indicates that there will be a significant difference in the predicted specific-impulse values based on which Δh_{turb} assumption is used.

Turbine Specific Work Assuming a Frozen Turbine-Drive Gas

A third approach to predicting the enthalpy decrease of a chemically reacting gas mixture is to assume a nonreacting gas mixture ("frozen" flow) in the gas generator. With frozen flow *Cp* of the mixture is the mass fraction-averaged *Cp* of the constituents. The value of *Cp* for the mixture is multiplied by the maximum (isentropic) temperature decrease to define the enthalpy decrease. In general, applying this approach to nozzle exhaust velocity predicts exhaust velocities that are 5 to 10% less than the exhaust velocities based on the assumption of an equilibrium gas mixture. Measured exhaust velocities are usually between those predicted using these two approaches (see Ref. 14). Similarly, the assumption of frozen flow yields turbine specific work values that are less than those based on an equilibrium gas mixture. Thus the actual enthalpy decrease of the gas mixture as it expands in the turbine will generally be between the enthalpy drop values assuming frozen flow and equilibrium flow. This important difference between reacting and nonreacting flows also applies to monopropellants, like the decomposition products of hydrazine monopropellants because they are a chemically reactive gas mixture.

Using this frozen turbine-drive gas assumption, Eq. (11) is modified to obtain Eq. (22) for $\Delta h_{turb\text{-}FROZEN}$:

$$\Delta h_{turb\text{-}FROZEN} = \eta_{turb} Cp_{turb\text{-}FROZEN} Tgg\left[1 - (1/PR_{turb})^{(\gamma_{turb}-1)/\gamma_{turb}}\right] \tag{22}$$

The CEA code was used to generate the needed frozen *Cp* values as well as *Tgg* values required in Eq. (22) to calculate $\Delta h_{turb\text{-}FROZEN}$.

Figure 8 is a plot of $\Delta h_{turb\text{-}FROZEN}$ (labeled DELTA *h* turb-FROZEN in figure) as a function of gas-generator O/F for an O_2/H_2-powered ATR. Figure 8 shows that $\Delta h_{turb\text{-}FROZEN}$ maximizes at a gas-generator O/F of about 3 and is nearly identical to the maximum $\Delta h_{turb\text{-}EQUILIBRIUM}$ in Fig. 6.

Figure 9 shows $\Delta h_{turb\text{-}FROZEN}$ (labeled DELTA *h* turb-FROZEN in figure) as a function of gas-generator O/F for an

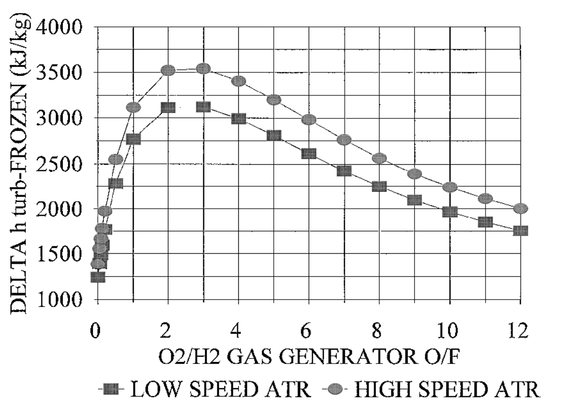

Fig. 8 $\Delta h_{turb\text{-}FROZEN}$ vs O_2/H_2 gas-generator O/F.

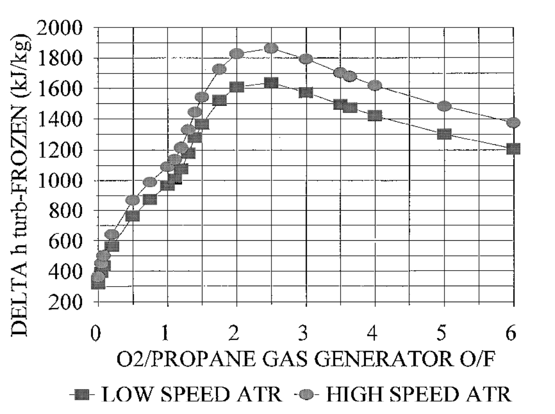

Fig. 9 $\Delta h_{turb\text{-}FROZEN}$ vs O_2/propane gas-generator O/F.

O_2/propane-powered ATR. In this figure $\Delta h_{turb\text{-}FROZEN}$ maximizes at a gas-generator O/F of about 2.5 for both ATR designs and is nearly identical to the maximum $\Delta h_{turb\text{-}EQUILIBRIUM}$ values in Fig. 7.

Comparison of Ideal and Equilibrium Turbine-Drive Gas Assumptions on Specific-Impulse Predictions

With the main combustor fuel-to-air ratio and *CpTcomb* determined for all three turbine-drive gas assumptions, the specific impulse corresponding to each assumption can now be calculated. Because the frozen and equilibrium assumptions yield very similar Δh_{turb} values, only the equilibrium and ideal gas expansion assumptions results are compared.

Figure 10 shows ATR specific impulse as a function of O_2/H_2 gas-generator O/F for both ATR designs for both the ideal and equilibrium turbine-drive assumptions. This figure shows two relative maximum specific-impulse values, at gas-generator O/F values of approximately 1 (depending on ATR design) and 6 for the ideal gas assumption. The first maximum at an O/F of about 1 corresponds to the maximum $\Delta h_{turb\text{-}IDEAL}$. The second maximum at an O/F of about 6 corresponds to the maximum *CpTcomb*. In contrast, this same figure shows that the specific impulse of both ATR designs maximize at a gas-generator O/F of about 3 for the equilibrium gas assumption. Most importantly, Fig. 10 shows that predicted specific-impulse values can be unrealistically high for gas-generator O/F values above 5 if an ideal gas is assumed. On the other hand, for O/F values below 5 the predicted specific impulse can be significantly low if an ideal turbine drive gas is assumed.

Figure 11 shows the specific impulse for the O_2/propane-powered ATR assuming ideal and equilibrium turbine-drive gas. As anticipated, there are two relative maximum specific-impulse values for ideal gas expansion. The first peak corresponds to the maximum $\Delta h_{turb\text{-}IDEAL}$ at an O/F of about 0.5 for both ATR designs. The second specific-impulse peak occurs at an O/F of about 3, which corresponds to the maximum *CpTcomb*. Figure 11 also shows that the ideal gas assumption results in predicted specific-impulse values considerably higher than those using the equilibrium gas assumption for almost all O/F values. Only at an O/F of about 1 do both assumptions yield similar specific-impulse values. Finally, Fig. 11 shows that, using equilibrium gas expansion in the turbine, there are two relative specific-impulse maximum values. These occur at an O/F of about 1 and 2, which correspond respectively to the maximum $\Delta h_{turb\text{-}EQUILIBRIUM}$ and *CpTcomb*.

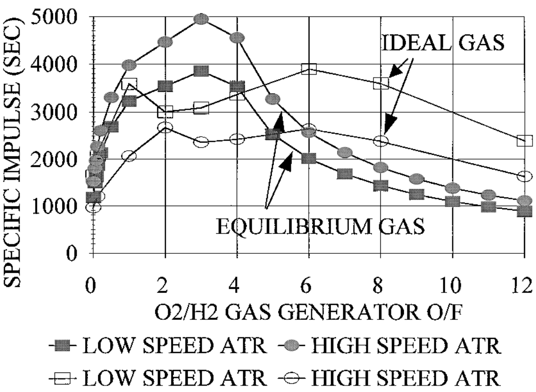

Fig. 10 Specific impulse vs O_2/H_2 gas-generator O/F.

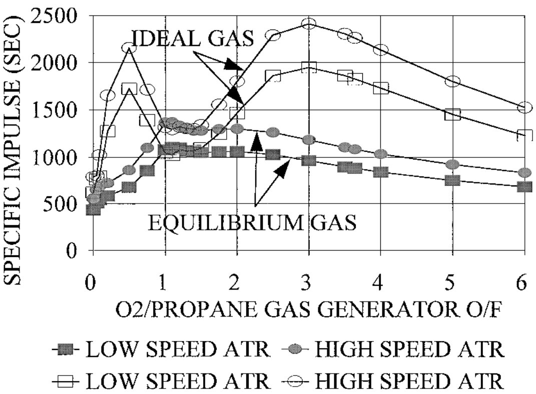

Fig. 11 Specific impulse vs O_2/propane gas-generator O/F.

Conclusions

The variation and magnitudes of predicted ATR specific impulse as a function of gas-generator O/F vary considerably depending on whether an ideal or equilibrium turbine gas is assumed. The major reason for this difference is that the turbine-drive gas can be a reacting gas mixture, which does not behave as an ideal gas. In particular, when using either O_2/H_2 or O_2/propane propellants in an ATR the turbine-drive gas should not be assumed to behave ideally. In fact, use of the ideal turbine-drive gas assumption can lead to a mistaken idea that there are two relative maximum specific-impulse values when using either of these propellant combinations. In the case of the O_2/H_2-driven ATR, there is only one design point maximum specific-impulse value, which occurs at a gas-generator O/F of about 3 for both low- and high-speed ATR designs. For the O_2/propane-driven ATR there are two relative maximum specific-impulse values possible. These maxima occur at gas-generator O/F values of about 1 and 2 and are nearly equal in magnitude.

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